



MARKSCHEME

November 2012

MATHEMATICS

Standard Level

Paper 2

16 pages

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Instructions to Examiners

Note: Changes linked to e-marking are noted in **red**. Other marking changes since November 2011 are noted in **green**. In particular, please note the removal of the accuracy **and misread** penalties and the revised accuracy instructions.

Abbreviations

- M** Marks awarded for attempting to use a correct **Method**; working must be seen.
- (M)** Marks awarded for **Method**; may be implied by **correct** subsequent working.
- A** Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding **M** marks.
- (A)** Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.
- R** **Marks awarded for clear Reasoning.**
- N** Marks awarded for **correct** answers if **no** working shown.
- AG** Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to scoris instructions and the document “**Mathematics SL: Guidance for e-marking November 2012**”. It is **essential** that you read this document before you start marking. In particular, please note the following. **Marks must be recorded using the annotation stamps, using the new scoris assessor marking tool. Please check that you are entering marks for the right question.**

- If a part is **completely correct**, (and gains all the “must be seen” marks), use the ticks with numbers to stamp full marks.
- If a part is completely wrong, stamp **A0** by the final answer.
- If a part gains anything else, it **must** be recorded using **all** the annotations.

All the marks will be added and recorded by scoris.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award **M0** followed by **AI**, as **A** mark(s) depend on the preceding **M** mark(s), if any. An exception to this rule is when work for **MI** is missing, as opposed to incorrect (see point 4).
- Where **M** and **A** marks are noted on the same line, *e.g.* **MIAI**, this usually means **MI** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and **AI** for using the **correct** values.
- Where there are two or more **A** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award **A0AIAI**.
- Where the markscheme specifies **(M2)**, **N3**, *etc.*, do **not** split the marks, unless there is a note.
- Once a correct answer to a question or part-question is seen, ignore further working.

3 ***N* marks**

If *no working shown*, award *N* marks for **correct** answers. In this case, ignore mark breakdown (*M*, *A*, *R*).

- Do **not** award a mixture of *N* and other marks.
- There may be fewer *N* marks available than the total of *M*, *A* and *R* marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.
- There may not be a direct relationship between the *N* marks and the implied marks. There are times when all the marks are implied, but the *N* marks are not the full marks: this indicates that we want to see some of the working, without specifying what.
- For consistency within the markscheme, *N* marks are noted for every part, even when these match the mark breakdown.
- If a candidate has incorrect working, which somehow results in a correct answer, do **not** award the *N* marks for this correct answer. However, if the candidate has indicated (usually by crossing out) that the working is to be ignored, award the *N* marks for the correct answer.

4 **Implied and must be seen marks**

Implied marks appear in brackets e.g. (MI).

- Implied marks can only be awarded if **correct** work is seen or if implied in subsequent working (a correct answer does not necessarily mean that the implied marks are all awarded). There are questions where some working is required, but as it is accepted that not everyone will write the same steps, all the marks are implied, but the *N* marks are not the full marks for the question.
- Normally the correct work is seen or implied in the next line.
- Where there is an (*MI*) followed by *AI* for each correct answer, if no working shown, one correct answer is sufficient evidence to award the (*MI*).

Must be seen marks appear without brackets e.g. MI.

- Must be seen marks can only be awarded if **correct** work is seen.
- If a must be seen mark is not awarded because work is missing (as opposed to *M0* or *A0* for incorrect work) all subsequent marks may be awarded if appropriate.

5 **Follow through marks (only applied after an error is made)**

*Follow through (FT) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the answer (i.e. there is no working expected), then **FT** marks should be awarded if appropriate. *Examiners are expected to check student work in order to award FT marks where appropriate.**

- Within a question part, once an **error** is made, no further *A* marks can be awarded for work which uses the error, but *M* marks may be awarded if appropriate. (However, as noted above, if an *A* mark is not awarded because work is missing, all subsequent marks may be awarded if appropriate)
- Exceptions to this rule will be explicitly noted on the markscheme.
- If the question becomes much simpler because of an error then use discretion to award fewer **FT** marks.
- If the error leads to an inappropriate value (e.g. probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin \theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.

- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.
- In a “show that” question, if an error leads to not showing the required answer, there is a 1 mark penalty. Note that if the error occurs within the same subpart, the *FT* rules may result in further loss of marks.
- Where there are anticipated common errors, the *FT* answers are often noted on the markscheme, to help examiners. It should be stressed that these are not the only *FT* answers accepted.

6 Mis-read

If a candidate incorrectly copies information from the question, this is a mis-read (*MR*). A candidate should be penalized only once for a particular mis-read. Use the *MR* stamp to indicate that this is a misread. Do not award the first mark in the question, even if this is an *M* mark, but award all others (if appropriate) so that the candidate only loses one mark for the misread.

- If the question becomes much simpler because of the *MR*, then use discretion to award fewer marks.
- If the *MR* leads to an inappropriate value (e.g. probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin \theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates’ own work does **not** constitute a misread, it is an error.

7 Discretionary marks (*d*)

An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation *DM* should be used and a brief **note** written next to the mark explaining this decision.

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, etc.
- Alternative solutions for part-questions are indicated by **EITHER . . . OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

10 Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures

Candidates should NO LONGER be penalized for an accuracy error (AP). Examiners should award marks according to the rules given in these instructions and the markscheme. Accuracy is not the same as correctness – an incorrect value does not achieve relevant A marks. It is only final answers which may lose marks for accuracy errors, not intermediate values. Please check work carefully for FT. Further information on which answers are accepted is given in a separate booklet, along with examples. It is essential that you read this carefully, as there are a number of changes.

Do not accept unfinished numerical answers such as $3/0.1$ (unless otherwise stated). As a rule, numerical answers with more than one part (such as fractions) should be given using integers (e.g. $6/8$). Calculations which lead to integers should be completed, with the exception of fractions which are not whole numbers.

11 Calculators

A GDC is required for paper 2, but calculators with symbolic manipulation features (e.g. TI-89) are not allowed.

Calculator notation

The Mathematics SL guide says:

Students must always use correct mathematical notation, not calculator notation.

Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

12 Style

The markscheme aims to present answers using good communication, e.g. if the question asks to find the value of k , the markscheme will say $k = 3$, but the marks will be for the correct value 3 – there is usually no need for the “ $k =$ ”. In these cases, it is also usually acceptable to have another variable, as long as there is no ambiguity in the question, e.g. if the question asks to find the value of p and of q , then the student answer needs to be clear. Generally, the only situation where the full answer is required is in a question which asks for equations – in this case the markscheme will say “must be an equation”.

The markscheme often uses words to describe what the marks are for, followed by examples, using the e.g. notation. These examples are not exhaustive, and examiners should check what candidates have written, to see if they satisfy the description. Where these marks are M marks, the examples may include ones using poor notation, to indicate what is acceptable.

13 Candidate work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on lined paper. Sometimes, they need more room for Section A, and use lined paper (and often comment to this effect on the QP), or write outside the box. That is fine, and this work should be marked.

The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the lined paper, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on the lined paper, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on the lined paper.

14. Diagrams

The notes on how to allocate marks for sketches usually refer to passing through particular points or having certain features. These marks can only be awarded if the sketch is approximately the correct shape. All values given will be an approximate guide to where these points/features occur. In some questions, the first **AI** is for the shape, in others, the marks are only for the points and/or features. In both cases, unless the shape is approximately correct, no marks can be awarded. However, if the graph is based on previous calculations, **FT** marks should be awarded if appropriate.

SECTION A

1. (a) valid method **(M1)**
 e.g. subtracting terms, using sequence formula
- $d = 1.7$ **A1** **N2**
[2 marks]
- (b) correct substitution into term formula **(A1)**
 e.g. $5 + 27(1.7)$
- 28^{th} term is 50.9 (exact) **A1** **N2**
[2 marks]
- (c) correct substitution into sum formula **(A1)**
 e.g. $S_{28} = \frac{28}{2}(2(5) + 27(1.7)), \frac{28}{2}(5 + 50.9)$
- $S_{28} = 782.6$ (exact) [782, 783] **A1** **N2**
[2 marks]
Total [6 marks]

2. (a) $A^{-1} = \begin{pmatrix} -0.5 & 0 & 0.5 \\ 1.5 & 1 & -1.5 \\ -1 & -2 & 2 \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} -\frac{1}{2} & 0 & \frac{1}{2} \\ \frac{3}{2} & 1 & -\frac{3}{2} \\ -1 & -2 & 2 \end{pmatrix} \end{pmatrix}$ **A2** **N2**

Note: Award **A1** for 6, 7 or 8 correct elements.

- (b) evidence of multiplying **AB** by A^{-1} (on left or right) **(M1)**
 e.g. 6, 7 or 8 correct elements **[2 marks]**
- $B = \begin{pmatrix} 3 & -2 & 4 \\ -4 & 5 & -9 \\ 1 & 0 & 9 \end{pmatrix}$ **A2** **N3**

Notes: Award **A1** for 6, 7 or 8 correct elements.
 Award **MIA1** if correct answer follows from working where matrices are written in reversed order.

[3 marks]
Total [5 marks]

3. (a) $x = 2$ (accept $(2, 0)$) AI N1
[1 mark]
- (b) evidence of finding gradient of f at $x = 2$ (M1)
e.g. $f'(2)$
 the gradient is 10 AI N2
[2 marks]
- (c) evidence of negative reciprocal of gradient (M1)
e.g. $\frac{-1}{f'(x)}, -\frac{1}{10}$
 evidence of correct substitution into equation of a line (A1)
e.g. $y - 0 = \frac{-1}{10}(x - 2), 0 = -0.1(2) + b$
 $y = -\frac{1}{10}x + \frac{2}{10}$ (accept $a = -0.1, b = 0.2$) AI N2
[3 marks]
- Total [6 marks]**

4. attempt to expand binomial (M1)
e.g. $(2x)^6 p^0 + \binom{6}{1}(2x)^5 (p)^1 + \dots, \binom{n}{r}(2x)^r (p)^{n-r}$
 one correct calculation for term in x^4 in the expansion for power 6 (A1)
e.g. $15, 16x^4$
 correct expression for term in x^4 (A1)
e.g. $\binom{6}{2}(2x)^4 (p)^2, 15.2^4 p^2$

Notes: Accept sloppy notation *e.g.* omission of brackets around $2x$.
 Accept absence of x in middle factor.

- correct term (A1)
e.g. $240p^2x^4$ (accept absence of x^4)
 setting up equation with **their** coefficient equal to 60 M1
e.g. $\binom{6}{2}(2)^4 (p)^2 = 60, 240p^2x^4 = 60x^4, p^2 = \frac{60}{240}$
 $p = \pm\frac{1}{2}$ ($p = \pm 0.5$) AIAI N3
[7 marks]

5. (a) (i) $a = 5$ (accept -5) A1 N1
 (ii) $c = 3$ (accept $c = 7$, if $a = -5$) A1 N1

Note: Accept other correct values of c , such as 11, -5 , etc.

[2 marks]

- (b) attempt to find period (M1)

e.g. $8, b = \frac{2\pi}{\text{period}}$

0.785398...

$b = \frac{2\pi}{8}$ (exact), $\frac{\pi}{4}$, 0.785 [0.785, 0.786] (do not accept 45) A1 N2

[2 marks]

- (c) valid approach (M1)

e.g. $f(x) = 0$, symmetry of curve

$x = 5$ (accept (5, 0)) A1 N2

[2 marks]

Total [6 marks]

6. correct z -values (A1)(A1)
 $-1.750686\dots, 0.524400\dots$

attempt to set up **their** equations, must involve z -values, not % (M1)

e.g. one correct equation

two correct equations A1A1

e.g. $\mu - 1.750686\sigma = 5, 0.5244005 = \frac{25 - \mu}{\sigma}$

attempt to solve **their** equations (M1)

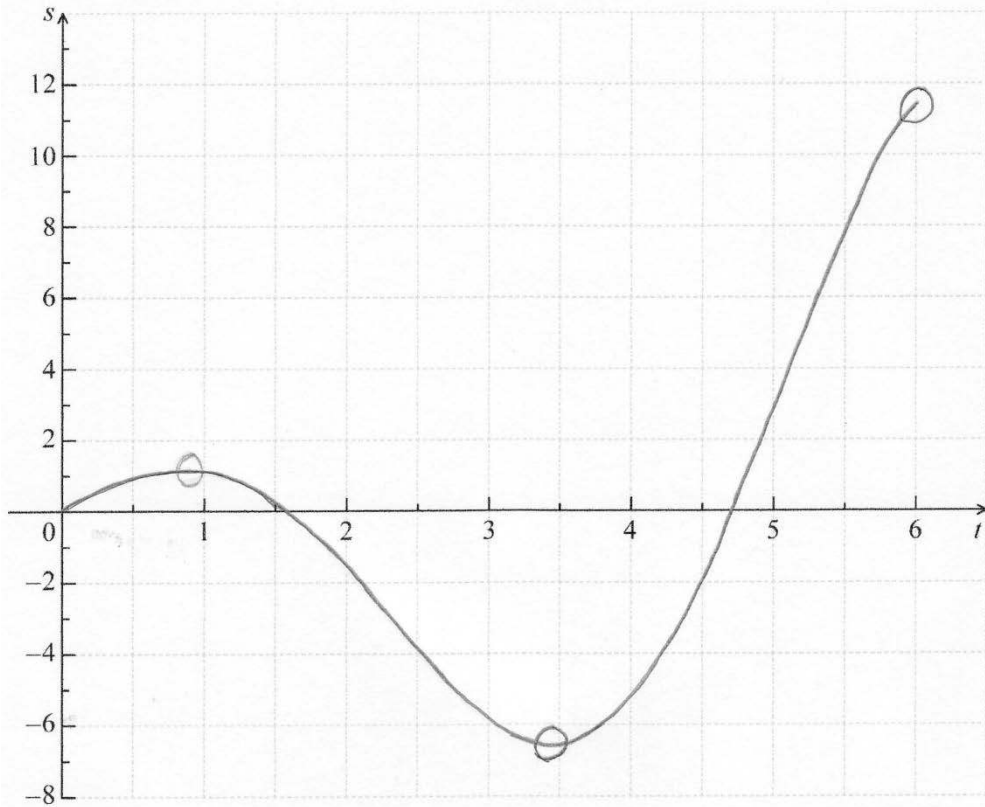
e.g. substitution, matrices, one correct value

$\mu = 20.39006\dots, \sigma = 8.790874\dots$

$\mu = 20.4$ [20.3, 20.4], $\sigma = 8.79$ [8.79, 8.80] A1A1 N4

[8 marks]

7. (a)



AIAIAIAI

N4

Note: Award *AI* for approximately correct shape (do not accept line segments).
Only if this *AI* is awarded, award the following:
AI for maximum and minimum within circles,
AI for *x*-intercepts between 1 and 2 **and** between 4 and 5,
AI for left endpoint at (0, 0) and right endpoint within circle.

[4 marks]

(b) appropriate approach (M1)

e.g. recognizing that $v = s'$, finding derivative, $a = s''$

valid method to find maximum (M1)

e.g. sketch of v , $v'(t) = 0$, $t = 5.08698\dots$

$v = 10.20025\dots$

$v = 10.2$ [10.2, 10.3]

AI

N2

[3 marks]

[7 marks]

SECTION B

8. **Note:** In this question, do not penalise for missing or incorrect units. They are not included in the markscheme, to avoid complex answer lines.

(a) **METHOD 1**

choosing cosine rule (must have cos in it) **(M1)**

e.g. $c^2 = a^2 + b^2 - 2ab \cos C$

correct substitution (into rhs) **A1**

e.g. $20^2 + 20^2 - 2(20)(20)\cos 1.5$, $AB = \sqrt{800 - 800\cos 1.5}$

$AB = 27.26555\dots$

$AB = 27.3$ [27.2, 27.3] **A1** **N2**
[3 marks]

METHOD 2

choosing sine rule **(M1)**

e.g. $\frac{\sin A}{a} = \frac{\sin B}{b}$, $\frac{AB}{\sin O} = \frac{AO}{\sin B}$

correct substitution **A1**

e.g. $\frac{AB}{\sin 1.5} = \frac{20}{\sin(0.5(\pi - 1.5))}$

$AB = 27.26555\dots$

$AB = 27.3$ [27.2, 27.3] **A1** **N2**
[3 marks]

(b) correct substitution into area formula **A1**

e.g. $\frac{1}{2}(20)(20)\sin 1.5$, $\frac{1}{2}(20)(27.2655504\dots)\sin(0.5(\pi - 1.5))$

area = 199.498997... (accept 199.75106 = 200, from using 27.3)

area = 199 [199, 200] **A1** **N1**
[2 marks]

continued ...

Question 8 continued

- (c) appropriate method to find angle AOC (MI)
e.g. $2\pi - 1.5 - 2.4$
- correct substitution into arc length formula (AI)
e.g. $(2\pi - 3.9) \times 20$, $2.3831853... \times 20$
- arc length = 47.6637...
- arc length = 47.7 (47.6, 47.7] (*i.e.* do **not** accept 47.6) AI N2

Notes: Candidates may misread the question and use $\hat{AOC} = 2.4$. If working shown, award **M0** then **A0MRAI** for the answer 48. Do not then penalize \hat{AOC} in part (d) which, if used, leads to the answer 679.498...

However, if they use the prematurely rounded value of 2.4 for \hat{AOC} , penalise 1 mark for premature rounding for the answer 48 in (c). Do not then penalize for this in (d).

[3 marks]

- (d) calculating sector area using **their** angle AOC (AI)
e.g. $\frac{1}{2}(2.38...)(20^2)$, $200(2.38...)$, $476.6370614...$
- shaded area = **their** area of triangle AOB + **their** area of sector (MI)
e.g. $199.4989973... + 476.6370614...$, $199 + 476.637$
- shaded area = 676.136... (accept 675.637... = 676 from using 199)
- shaded area = 676 [676, 677], AI N2

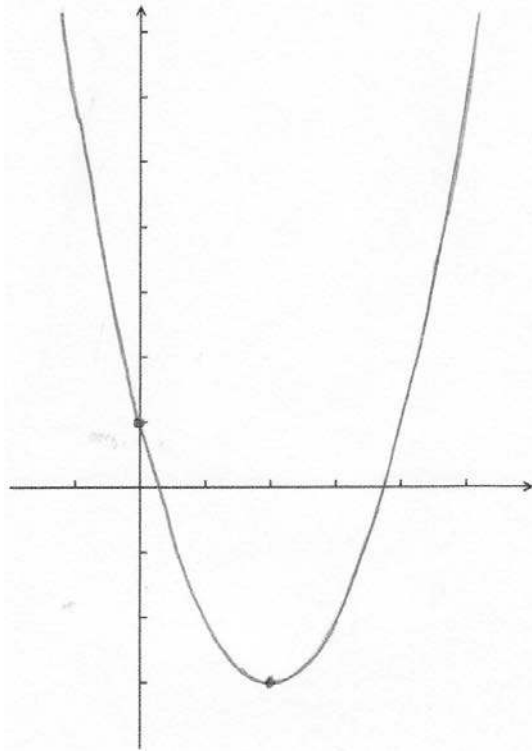
[3 marks]

- (e) dividing to find number of cans (MI)
e.g. $\frac{676}{140}$, $4.82857...$
- 5 cans must be purchased (AI)
- multiplying to find cost of cans (MI)
e.g. $5(32)$, $\frac{676}{140} \times 32$
- cost is 160 (dollars) AI N3

[4 marks]

Total [15 marks]

9. (a)



AIAIAIAI

N4

Note: The shape **must** be an approximately correct upwards parabola.
Only if the shape is approximately correct, award the following:
 AI for vertex $x \approx 2$, AI for x-intercepts between 0 and 1, and 3 and 4,
 AI for correct y-intercept (0, 1), AI for correct domain. [-1, 5].
 Scale not required on the axes, but approximate positions need to be clear.

[4 marks]

(b) $p = 2$

AI

N1

[1 mark]

(c) correct vertical reflection, correct vertical translation
 e.g. $-f(x)$, $-((x-2)^2 - 3)$, $-y$, $-f(x) + 6$, $y + 6$

(AI)(AI)

transformations in correct order

(AI)

e.g. $-(x^2 - 4x + 1) + 6$, $-((x-2)^2 - 3) + 6$

simplification which clearly leads to given answer

AI

e.g. $-x^2 + 4x - 1 + 6$, $-(x^2 - 4x + 4 - 3) + 6$

$$g(x) = -x^2 + 4x + 5$$

AG

N0

Note: If working shown, award AIAIA0A0 if transformations correct, but done in reverse order, e.g. $-(x^2 - 4x + 1) + 6$.

[4 marks]

continued ...

Question 9 continued

- (d) valid approach (MI)
 e.g. sketch, $f = g$

$-0.449489\dots, 4.449489\dots$

$(2 \pm \sqrt{6})$ (exact), $-0.449[-0.450, -0.449]$; $4.45[4.44, 4.45]$ AIAI N3
[3 marks]

- (e) attempt to substitute limits or functions into area formula (accept absence of dx) (MI)

e.g. $\int_a^b ((-x^2 + 4x + 5) - (x^2 - 4x + 1))dx, \int_{4.45}^{-0.449} (f - g),$
 $\int(-2x^2 + 8x + 4) dx$

approach involving subtraction of integrals/areas (accept absence of dx) (MI)

e.g. $\int_a^b (-x^2 + 4x + 5) - \int_a^b (x^2 - 4x + 1), \int (f - g) dx$

area = 39.19183...

area = 39.2 [39.1, 39.2] AI N3
[3 marks]

Total [15 marks]

10. (a) valid approach (MI)
 e.g. Venn diagram with intersection, union formula,
 $P(S \cap F) = 0.75 + 0.40 - 1$
 15 (accept 15 %) AI N2
 [2 marks]
- (b) valid approach involving subtraction (MI)
 e.g. Venn diagram, $75 - 15$
 60 (accept 60 %) AI N2
 [2 marks]
- (c) (i) valid approach (MI)
 e.g. tree diagram, multiplying probabilities, $P(S | G) \times P(G)$
 correct calculation (AI)
 e.g. 0.52×0.85
 $P(G \cap S) = 0.442$ (exact) AI N3
- (ii) valid reasoning, with words, symbols or numbers (seen anywhere) RI
 e.g. $P(G) \times P(S) \neq P(G \cap S)$, $P(S | G) \neq P(S)$, not equal,
 one correct value AI
 e.g. $P(G) \times P(S) = 0.39$, $P(S | G) = 0.85$, $0.39 \neq 0.442$
 G and S are not independent AG N0
 [5 marks]
- (d) **METHOD 1**
 48 % are boys (seen anywhere) AI
 e.g. $P(B) = 0.48$
 appropriate approach (MI)
 e.g. $P(\text{girl and Spanish}) + P(\text{boy and Spanish}) = P(\text{Spanish})$
 correct approach to find $P(\text{boy and Spanish})$ (AI)
 e.g. $P(B \cap S) = P(S) - P(G \cap S)$, $P(B \cap S) = P(S | B) \times P(B)$, 0.308
 correct substitution (AI)
 e.g. $0.442 + 0.48x = 0.75$, $0.48x = 0.308$
 correct manipulation (AI)
 e.g. $P(S | B) = \frac{0.308}{0.48}$
 $P(\text{Spanish} | \text{boy}) = 0.641666\dots, 0.641\bar{6}$
 $P(\text{Spanish} | \text{boy}) = 0.642$ [0.641, 0.642] AI N3
 [6 marks]

continued ...

Question 10 continued

METHOD 2

48 % are boys (seen anywhere) *AI*
e.g. 0.48 used in tree diagram

appropriate approach *(AI)*
e.g. tree diagram

correctly labelled branches on tree diagram *(AI)*
e.g. first branches are boy/girl, second branches are Spanish/not Spanish

correct substitution *(AI)*
e.g. $0.442 + 0.48x = 0.75$

correct manipulation *(AI)*
e.g. $0.48x = 0.308$, $P(S | B) = \frac{0.308}{0.48}$

$P(\text{Spanish} | \text{boy}) = 0.641666\dots, 0.641\bar{6}$

$P(\text{Spanish} | \text{boy}) = 0.642 [0.641, 0.642]$ *AI* *N3*
[6 marks]

Total [15 marks]